تم الرفع بواسطة
م. معن ابو عيسى

Calculus 1
Second

33/60

Palestine Technical University
Mathematics Department.
Calculus I
Dr. Khalida Nazzál

Student # [redacted] **Second Exam** Dec. 7th, 2009
Student name بالعربية: Sec #: RTN 11:00 - 12:00
معن سلام قعدان

Part I: Choose the correct answer (30 points)

1. The displacement of a particle moving in a straight line is given by $s = \frac{t^4}{4} - 8t + 3$.

When the velocity of the particle is equal to zero.
(a) $t = 4$
(b) $t = 0$
(c) $t = 1$
(d) $t = 2$

$s =$ $\frac{4t^3}{4} - 8$ $t^3 - 8 = 0$
$t^3 = 8 \Rightarrow t = \sqrt[3]{8} = 2$

2. If the sides of a cube changes from 2 to 2.01, the new volume is approximately
(a) 0.12.
(b) 8.12
(c) 8.00
(d) 7.88.

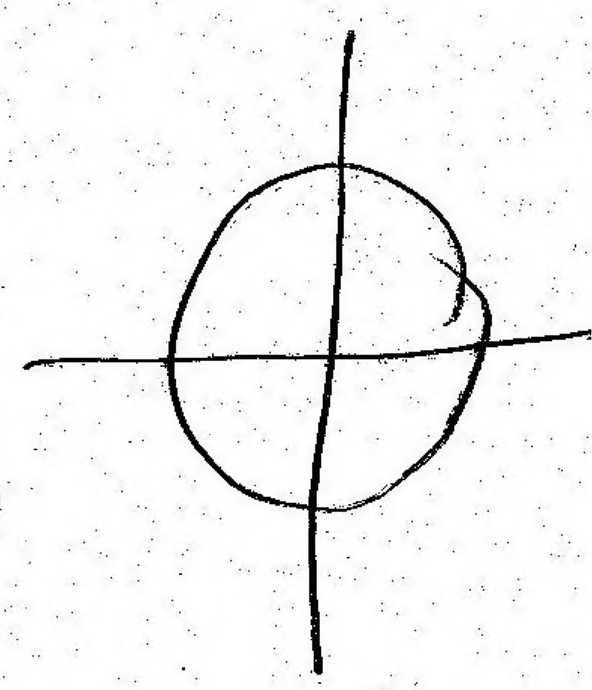
$dx = 0.01$ $8 + 0.12 = 8.12$
$V = x^3 = 2^3 = 8$
$dV = 3x^2 dx = 3(2)^2(0.01)$ 0.12

3. The equation of the tangent line to the curve $x^3 - xy + y^3 = 7$ at $P(2,1)$ is.
(a) $5(y-1) = 11(x-2)$.
(b) $y = 11x - 21$.
(c) $(y-1) = -11(x-2)$.
(d) else.

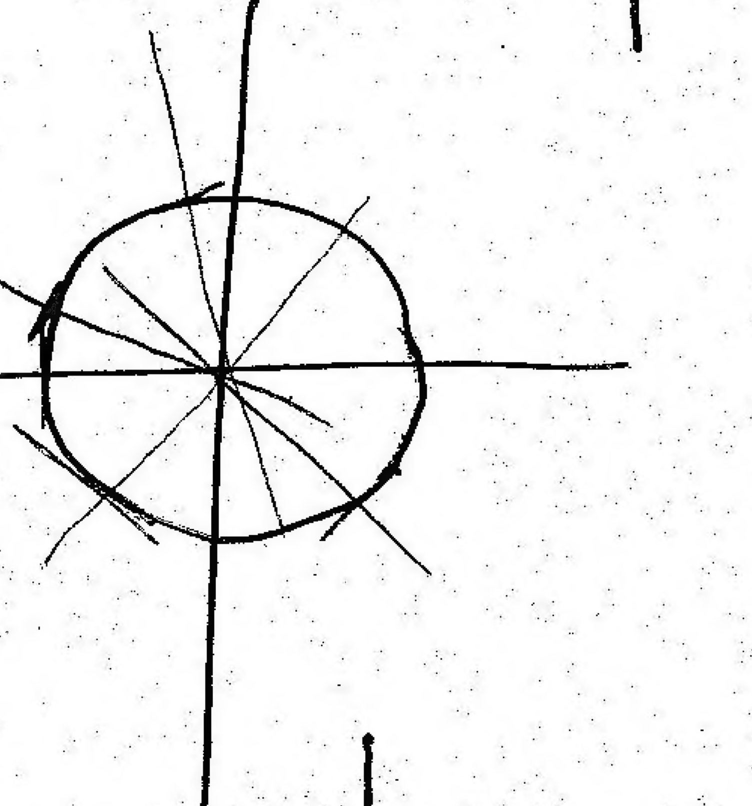
$3x^2 + x\frac{dy}{dx} + y + 3y^2\frac{dy}{dx} = 0$ $\frac{dy}{dx} = \frac{3(2)^2+1}{-2-3}$ $\frac{16}{-5}$ $\frac{13}{-5}$
$3x^2 + y = -x\frac{dy}{dx} - 3y\frac{dy}{dx}$
$y - 1 = \frac{18}{-5}(x-2)$ $3x^2 + y = -(x+3y)\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{3x^2+1}{-x-3y}$
$y - 1 = \frac{-16}{5}x + \frac{32}{5}$ $5y - 5 = -16x + 32$

4. Find the slope of the tangent line to the curve $x = 2 + \sin t$, $y = 3 + \cos t$, at $t = \frac{\pi}{4}$.
(a) 1
(b) 0
(c) -1
(d) not defined

$\frac{1}{\sqrt{2}} = x - 2$ $\sin t = x - 2$ $\frac{1}{\sqrt{2}}$ $(x-2)(x-2)$
$\frac{1}{\sqrt{2}} + \frac{2}{1}$ $\cos t = y - 3$ $x^2 - 2x - 2x + 4$
$\frac{2\sqrt{2}+1}{\sqrt{2}}$ $\sin^2 t + \cos^2 t = 1$ $x^2 - 4x + 4$
$(x-2)^2 + (y-3)^2 = 1$ $y^2 - 6y + 9$
$x^2 + 4x + 4 + y^2 - 6y + 9 = 1$

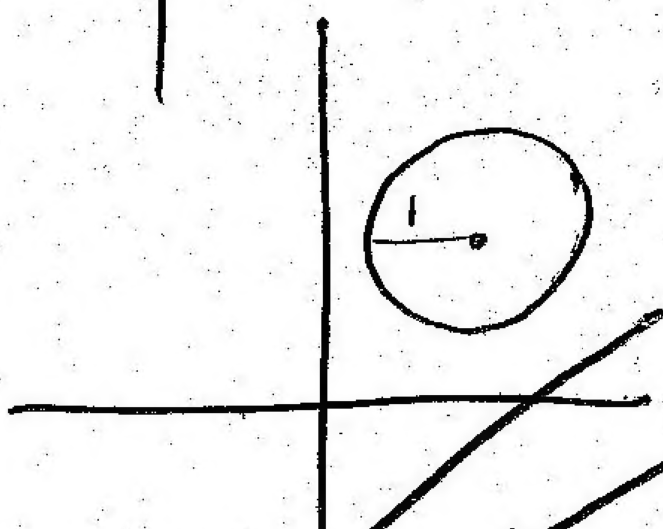


5. The normal line at any point of the circle $x^2 + y^2 = a^2$
(a) passes through the origin.
(b) has a negative slope
(c) is never a vertical line.
(d) is never a horizontal line.



$2(x-2) + 3(y-3)y' = 0$
$2x - 4 + 3yy' - 3y' = 0$
$2x - 4 = 3y' - 3yy'$
$y'(3 - 3y) = 2x - 4$
$y' = \frac{2x-4}{3-3y}$
$y' = \frac{2(\frac{1}{\sqrt{2}}+2) - 4}{3 - 3(\frac{1}{\sqrt{2}}+3)}$
$\frac{\sqrt{2}+4-4}{3 - \frac{3}{\sqrt{2}} - 9}$

$\frac{\frac{\sqrt{2}}{-6\sqrt{2}-3}}{\sqrt{2}}$
$\frac{2}{-6\sqrt{2}-3}$ $\frac{\sqrt{2}}{-6 - \frac{3}{\sqrt{2}}}$ $\frac{\sqrt{2}}{-6 - \frac{3}{\sqrt{2}}}$



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~~$x=-1$~~ $y=2$

6. Let x and y be functions of time t such that $x^2 + y^3 = 9$.

If $\frac{dx}{dt} = 5$, find $\frac{dy}{dt}$ when x=-1.

(a) $\frac{5}{8}$

(b) $\frac{5}{12}$

(c) $\frac{1}{2\sqrt{2}}$

(d) $\frac{5}{6}$.

$2x\frac{dx}{dt} + 3y^2\frac{dy}{dt} = 0$

$2(-1)(5) + 3y^2\frac{dy}{dt} = 0$

$10 = 3y^2\frac{dy}{dt}$

$x^2 = 9 - y^3$

$1 = 9 - y^3$

$y^3 = 9 - 1 = 8$

$y = 2$

$2(-1)(5) + 3(4)\frac{dy}{dt} = 0$

$-10 + 12\frac{dy}{dt} = 0$

$10 = 12\frac{dy}{dt} = \frac{10}{12}$ $\frac{5}{6}$

7. Given that $f(x) = 5 - \frac{4}{x}$, $x \in [2,4]$. Find the value(s) of c in the conclusion of the Mean Value Theorem.

(a) $2\sqrt{2}, -2\sqrt{2}$

(b) 1

(c) $2\sqrt{2}$

(d) 2,-2

$f'(c) = \frac{f(b) - f(a)}{b-a}$

$5 - 4x^{-1}$

$= 4x^{-2} = \frac{4}{x^2}$ $\quad \frac{4}{c^2} = \frac{4-3}{4-2} \Rightarrow \frac{4}{c^2} = \frac{1}{2}$ $\quad c^2 = 8$

$c = \pm\sqrt{8}$

$\pm 2\sqrt{2}$

8. If $h(x) = g(f(x))$ and $f(2)$=5, $g(5)$=3, $f'(2) = 7$, and $g'(5) = -2$, find $h'(2)$.

(a) -14

(b) 21

(c) -10

(d) 14.

$h'(2) = g'(f(2)) = g'(5)$

$h'(2) = g'(f(2))\,f'(2)$

$h'(2) = g'(5)\,(7) = -2\,(7) = -14$

9. A parametrization of the left half of the parabola $y = x^2 + 2x$ could be

(a) $x = t^2,\ y = t^4 + 2t^2,\ ,-\infty < t < \infty.$

(b) $x = t,\ y = t^2 + 2t,\ ,-\infty < t < \infty.$

(c) $x = \sin t,\ y = \sin^2 t + 2\sin t,\ ,-\infty < t < \infty.$

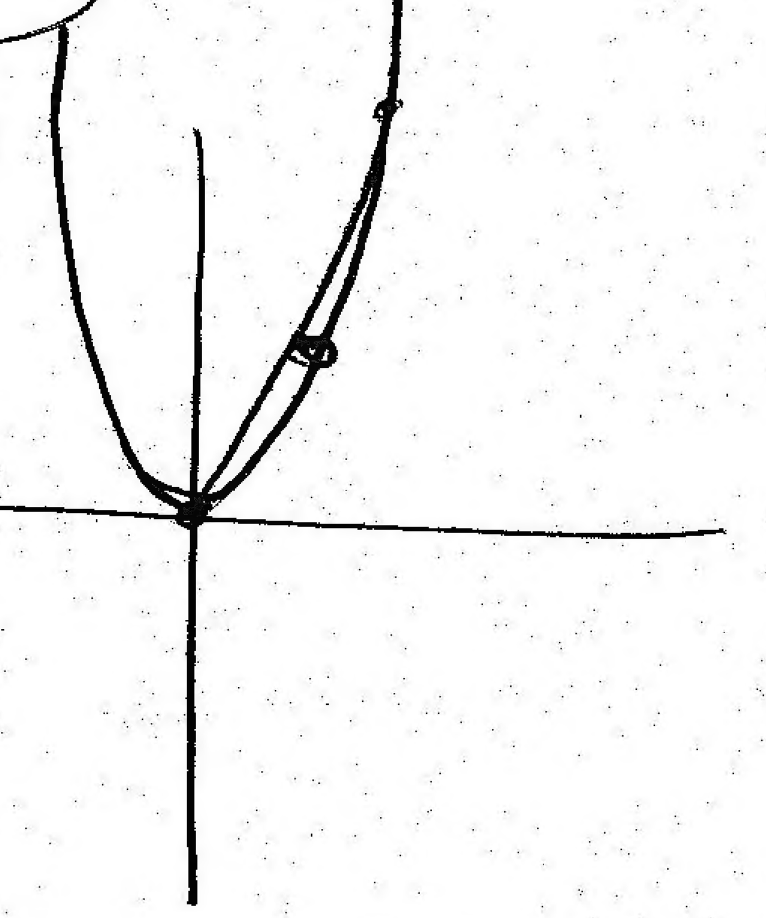
(d) $x = t,\ y = t^2 + 2t,\ ,t \le -1$

$x = t^2$

$y = (t^2)^2 + 2t^2$

$t^4 + 2t^2$

t^2



10. The linearization of $f(x) = \sqrt{x^2 + 8}$ at x = -1 is

(a) $y = \frac{8}{3} - \frac{1}{3}x$

(b) $y = 1 + \frac{1}{3}x$

(c) $y = 1 + \frac{1}{2}x$

(d) None of the above

$f(x_0) + f'(x_0)(x - x_0)$

$f(x) = (x^2 + 8)^{\frac{1}{2}}$ $\quad f'(x) = \frac{1}{2}(x^2+8)^{-\frac{1}{2}}(2x)$

$= \frac{x}{\sqrt{x^2+8}}$

$3 + \frac{-1}{3}(x+1)$

$\boxed{y = 2 - \frac{1}{3}x}$

$3 - \cancel{\frac{1}{3}x + 1}$

$3 - \frac{1}{3}(x+1)$

$3 - \frac{1}{3}x - 1$

$y = 2 - \frac{1}{3}x$

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$x^{\frac{1}{3}}$ $x^{\frac{2}{3}}$

II. (20 points). Let $f(x) = (\frac{5}{2} - x)\,x^{\frac{2}{3}}$.

(a) find the intervals on which f is increasing or decreasing.

$f'(x) = (\frac{5}{2}-x)\frac{2}{3}x^{-\frac{1}{3}} + x^{\frac{2}{3}}(-1) = \frac{10}{6}x^{-\frac{1}{3}} - \frac{2}{3}x^{\frac{2}{3}} - x^{\frac{2}{3}} =$

$\frac{5}{3\sqrt[3]{x}} - \frac{5}{3}\sqrt[3]{x^2} = \frac{5}{3}\left(\frac{1}{\sqrt[3]{x}} - \frac{\sqrt[3]{x^2}}{1}\right) = \frac{5}{3}\left(\frac{1-x}{\sqrt[3]{x}}\right) = 0$

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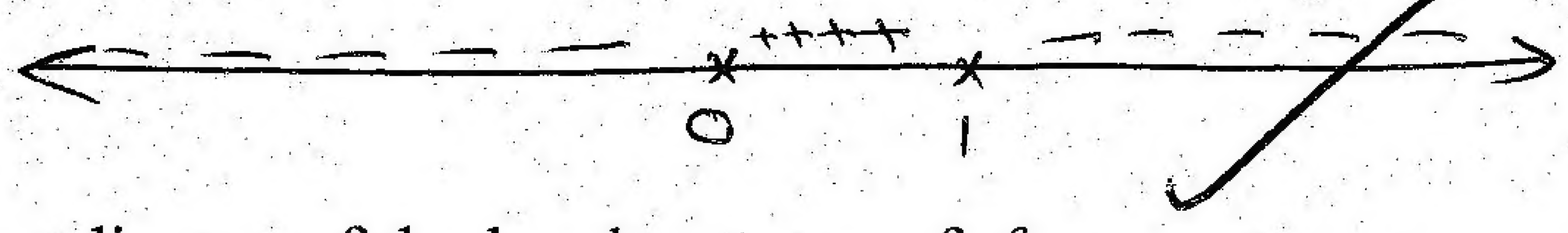
$\therefore \frac{1-x}{\sqrt[3]{x}} = 0 \Rightarrow \boxed{x=1}$, DNE $\boxed{x=0}$



increasing $0<x<1$

decreasing $x>1$, $x<0$ $(1,\infty)\cup(-\infty,0)$

(b) find the x-coordinates of the local extrema of f.

$f(0) = 0 \Rightarrow (0,0) \Rightarrow$ local minimum

4 $f(1) = \frac{3}{2}$, $(1, 1.5) \Rightarrow$ local maximum

$\frac{x^5}{x^4}$

(c) find the intervals on which f is CU or CD.

$f'(x) = \frac{5}{3}\left(\frac{1-x}{\sqrt[3]{x}}\right) \Rightarrow f''(x) = \frac{\sqrt[3]{x}(-1) - (1-x)(\frac{1}{3}x^{-\frac{1}{3}})}{x^{\frac{2}{3}}}$

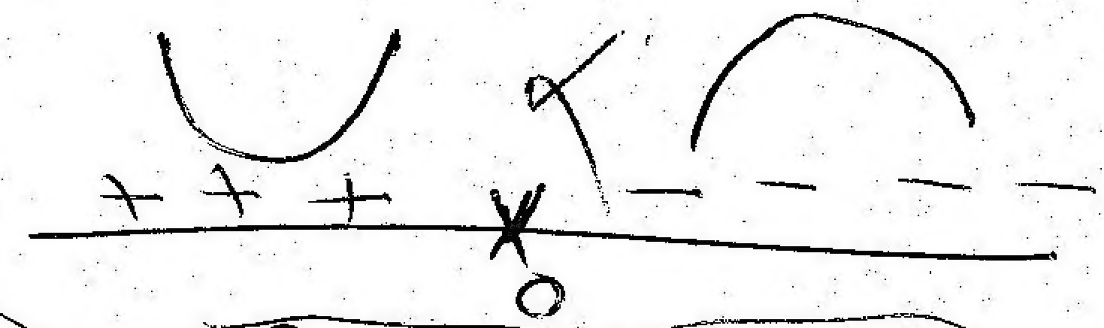
$= \frac{-\sqrt[3]{x} - [\frac{1}{3}x^{-\frac{1}{3}} - \frac{1}{3}x^{\frac{2}{3}}]}{x^{\frac{2}{3}}} = \frac{-\sqrt[3]{x} - \frac{1}{3}x^{-\frac{1}{3}} + \frac{1}{3}x^{\frac{2}{3}}}{x^{\frac{2}{3}}}$

$= \frac{-x^{\frac{1}{3}} - \frac{1}{3}x^{-\frac{1}{3}} + \frac{1}{3}x^{\frac{2}{3}}}{x^{\frac{2}{3}}} = -x^{-\frac{1}{3}} - \frac{1}{3}x^{-1} + \frac{1}{3} = 0$

$-\frac{1}{\sqrt[3]{x}} - \frac{1}{3x} + \frac{1}{3} = 0 \Rightarrow$ DNE at $x=0$

(d) find the x-coordinates of the points of inflection of f.

$f(0) = \cancel{0} \Rightarrow (0,0)$

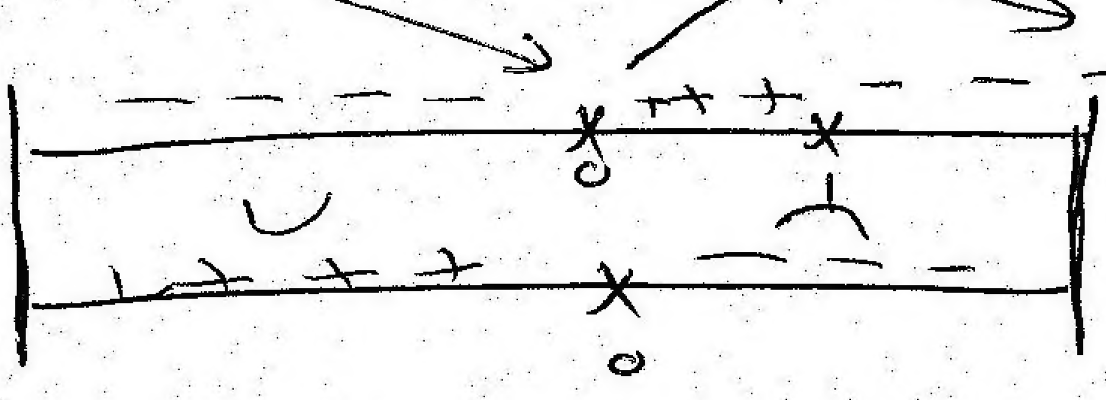


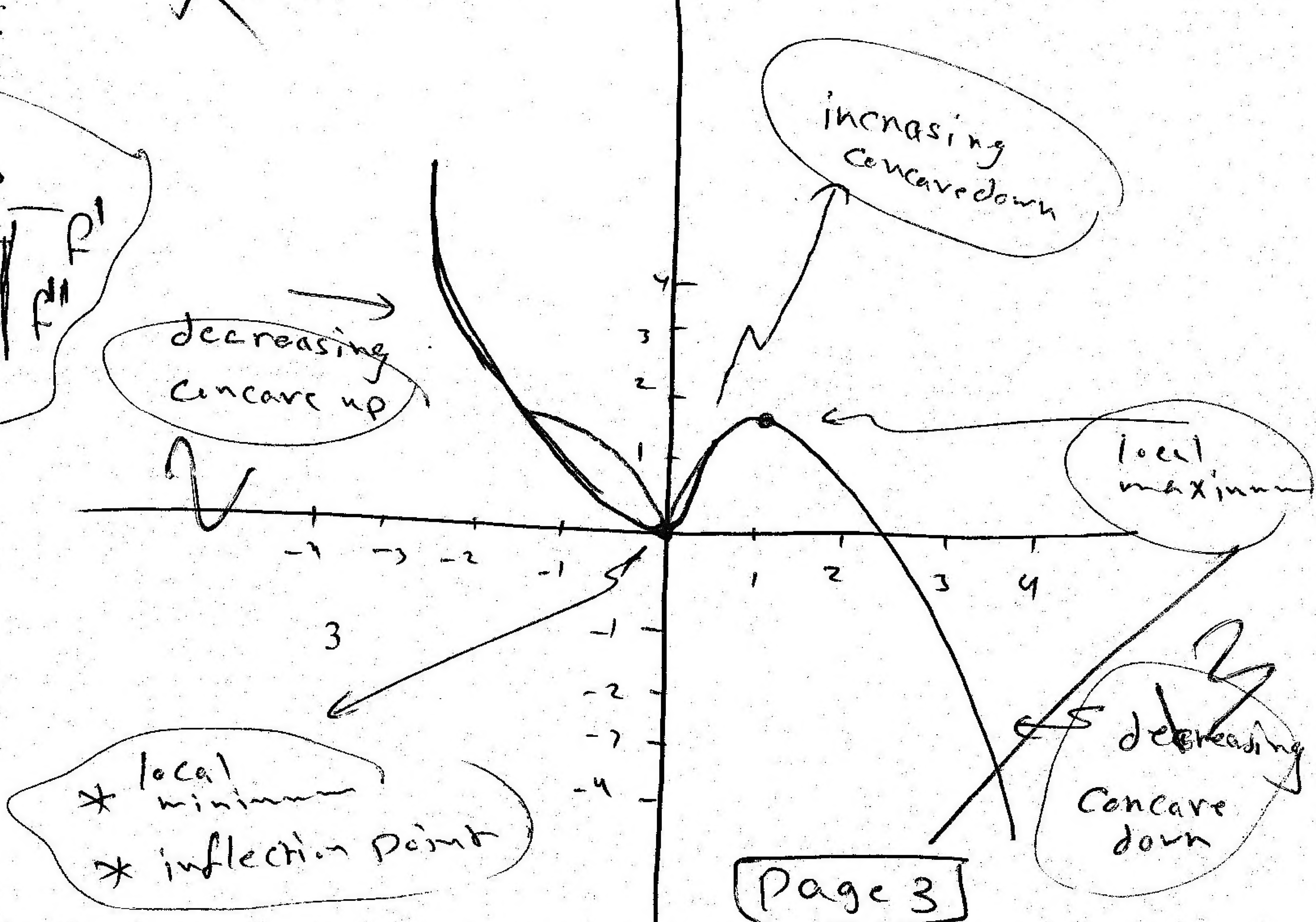
Concave up <0

Concave down >0

* find The cusp points i~~f exists~~

(e) Sketch the graph of f.





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III. A particle moves along the curve $y = x^{\frac{3}{2}}$ in the first quadrant in such a way that its distance from the origin increases at the rate 11 units per second. Find $\frac{dx}{dt}$ when $x = 3$.

(10 points).

$y = x^{\frac{3}{2}}$, $\frac{ds}{dt} = 11$

distance $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$\therefore$ distance $= \sqrt{x^2 + y^2} = \sqrt{x^2 + (x^{3/2})^2}$

when $x = 3 \Rightarrow y = 3^{\frac{3}{2}} = \sqrt[2]{3^3} = \boxed{\sqrt{27}}$

~~$(x^2 + y^2)^{\frac{1}{2}} \Rightarrow \frac{ds}{dt} = \frac{1}{2}(x^2 + y^2)^{-\frac{1}{2}}(2x\,dx + 2y\,dy)$~~

NO!

$\therefore S = \sqrt{x^2 + 27} = (x^2 + 27)^{\frac{1}{2}}$

$\therefore \frac{ds}{dt} = \frac{1}{2}(x^2 + 27)^{-\frac{1}{2}}(2x\,dx)$

$\therefore 11 = \frac{1}{2}(3^2 + 27)^{-\frac{1}{2}}(2(3)\,dx)$

$11 = \frac{1}{2\sqrt{36}} \cdot (6\,dx)$

$11 = \frac{1}{12} \cdot 6(dx)$

$11 = \frac{1}{2}(dx) \Rightarrow dx = \boxed{22 \text{ units/second}}$

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